Luenberger-Beobachter und Extended Kalman-Filter: Ein Vergleich

Hauptseminar Intelligente Verfahren in der Mechatronik

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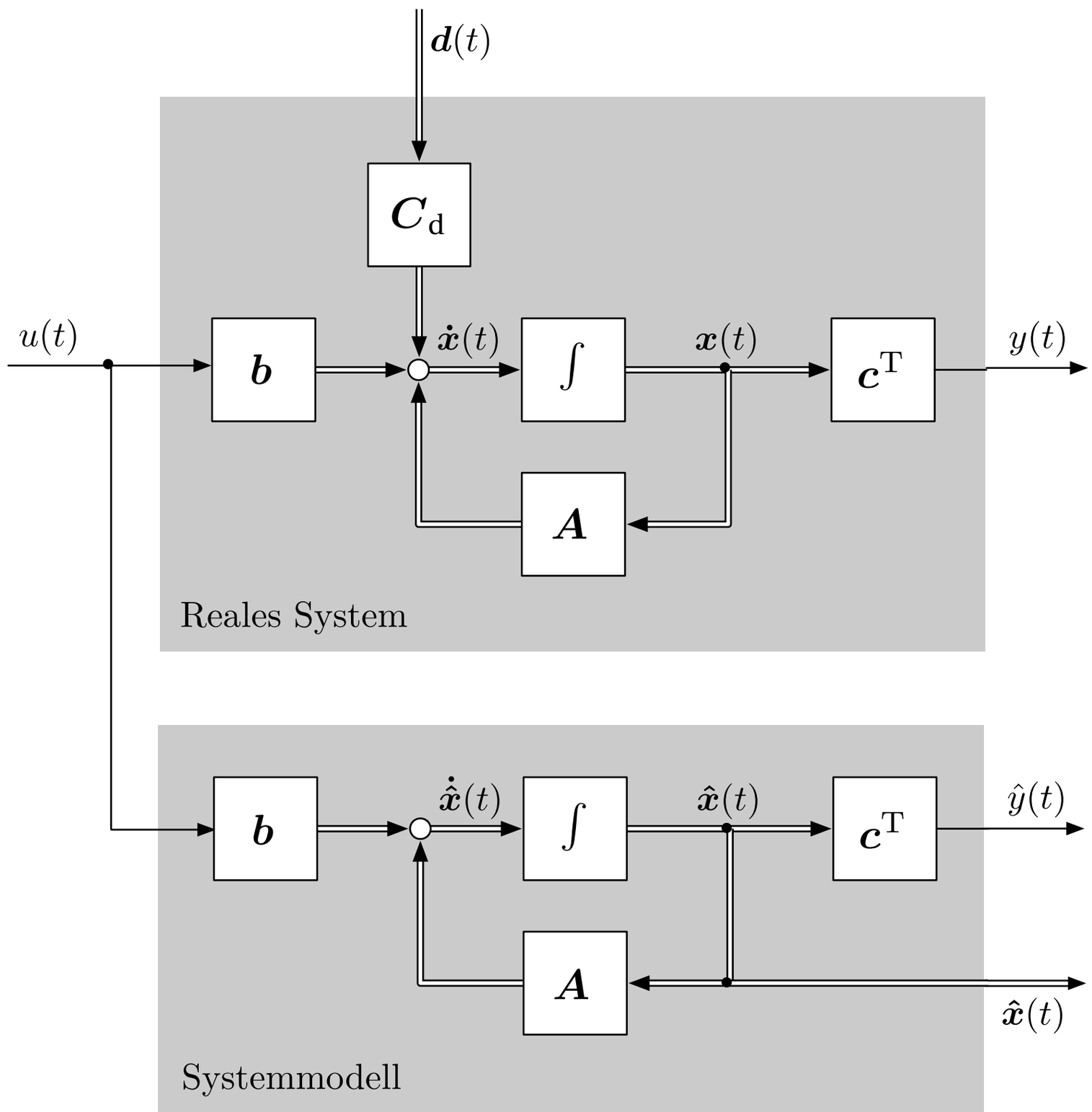
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$\boldsymbol{C}_\mathrm{d}$
$u(t)$
$\boldsymbol{b}$
$\dot{\boldsymbol{x}}(t)$
$\int$
$\boldsymbol{x}(t)$
$\boldsymbol{c}^\mathrm{T}$
$y(t)$
$\boldsymbol{A}$
Reales System
$\boldsymbol{b}$
$\dot{\hat{\boldsymbol{x}}}(t)$
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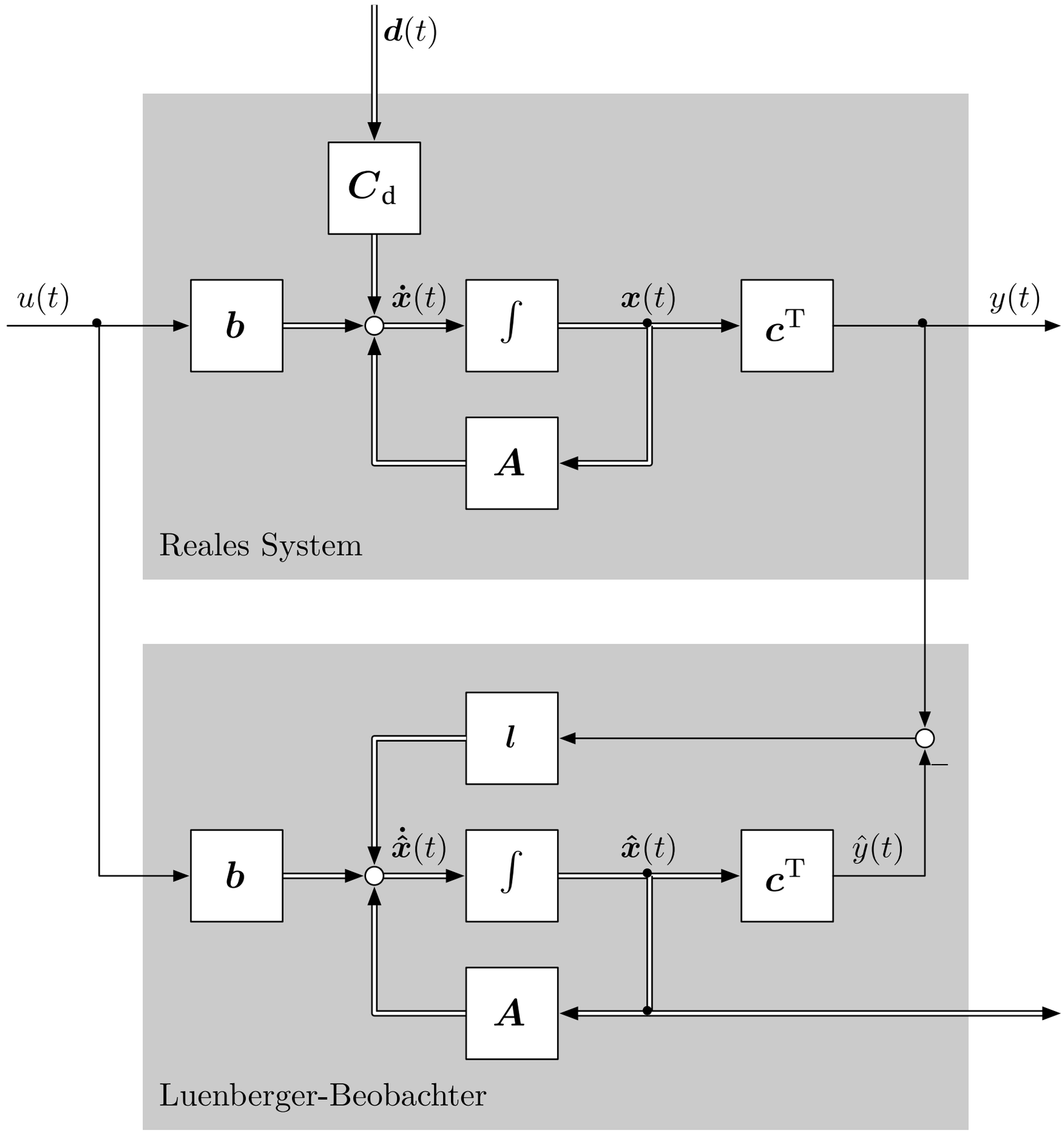
$\hat{\boldsymbol{x}}(t)$
$\boldsymbol{c}^\mathrm{T}$
$\hat{y}(t)$
$\boldsymbol{A}$
$\hat{\boldsymbol{x}}(t)$
Systemmodell


$\boldsymbol{d}(t)$
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$\dot{\hat{\boldsymbol{x}}}(t)$
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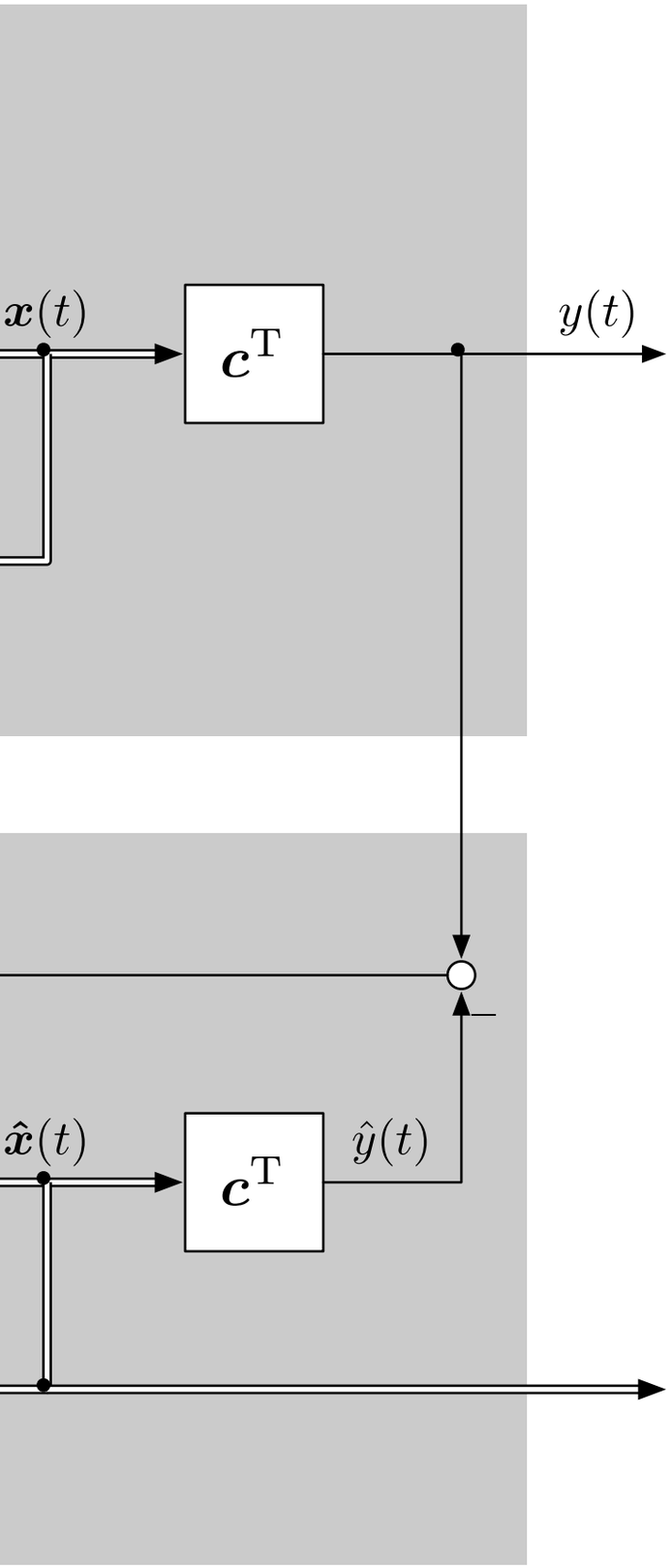
Luenberger-Beobachter



$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}u(t) + \boldsymbol{C}_d\boldsymbol{d}(t), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0$$
$$y(t) = \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}(t)$$

$$\Delta\boldsymbol{x}(t) = \boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t)$$
$$\Delta\dot{\boldsymbol{x}}(t) = \dot{\boldsymbol{x}}(t) - \dot{\hat{\boldsymbol{x}}}(t)$$
$$= \dots$$
$$= (\boldsymbol{A} - \boldsymbol{l}\boldsymbol{c}^{\mathrm{T}})\Delta\boldsymbol{x}(t) + \boldsymbol{C}_d\boldsymbol{d}(t) \xrightarrow{\text{stationär}} 0 = (\boldsymbol{A} - \boldsymbol{l}\boldsymbol{c}^{\mathrm{T}})\Delta\boldsymbol{x} + \boldsymbol{C}_{\mathrm{d}}\boldsymbol{d}$$

$$\dot{\hat{\boldsymbol{x}}}(t) = \boldsymbol{A}\hat{\boldsymbol{x}}(t) + \boldsymbol{b}u(t) + \boldsymbol{u}_{\mathrm{B}}(t), \quad \hat{\boldsymbol{x}}(0) = \hat{\boldsymbol{x}}_0$$
$$\hat{y}(t) = \boldsymbol{c}^{\mathrm{T}}\hat{\boldsymbol{x}}(t)$$

$$\boldsymbol{u}_{\mathrm{B}}(t) = \boldsymbol{l}(y(t) - \hat{y}(t)) = \boldsymbol{l}y(t) - \boldsymbol{l}\boldsymbol{c}^{\mathrm{T}}\hat{\boldsymbol{x}}(t)$$

3 Gleichungen des Luenberger-Beobachters

$$
\begin{aligned}
\dot{\boldsymbol{x}}_{\mathbf{d}}(t) &= \boldsymbol{A}_{\mathrm{d}}\boldsymbol{x}_{\mathrm{d}}(t) = \mathbf{0}\boldsymbol{x}_{\mathrm{d}}(t) = \mathbf{0}, \quad \boldsymbol{x}_{\mathrm{d}}(0) = \boldsymbol{x}_{\mathrm{d},0} = \boldsymbol{d}_{\infty} \\
\boldsymbol{y}_{\mathrm{d}}(t) &= \boldsymbol{C}_{\mathrm{d}}\boldsymbol{x}_{\mathrm{d}}(t) = \boldsymbol{C}_{\mathrm{d}}\boldsymbol{d}_{\infty}
\end{aligned}
$$

$$
\begin{aligned}
\begin{pmatrix} \dot{\boldsymbol{x}}(t) \\ \dot{\boldsymbol{x}}_{\mathrm{d}}(t) \end{pmatrix} &= \begin{pmatrix} \boldsymbol{A} & \boldsymbol{C}_{\mathrm{d}} \\ \mathbf{0} & \boldsymbol{A}_{\mathrm{d}} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}_{\mathrm{d}}(t) \end{pmatrix} + \begin{pmatrix} \boldsymbol{b} \\ \mathbf{0} \end{pmatrix} u(t) \\
y(t) &= \begin{pmatrix} \boldsymbol{c}^{\mathrm{T}} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}_{\mathrm{d}}(t) \end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
\begin{pmatrix} \dot{\hat{\boldsymbol{x}}}(t) \\ \dot{\hat{\boldsymbol{x}}}_{\mathrm{d}}(t) \end{pmatrix} &= \begin{pmatrix} \boldsymbol{A} & \boldsymbol{C}_{\mathrm{d}} \\ \mathbf{0} & \boldsymbol{A}_{\mathrm{d}} \end{pmatrix} \begin{pmatrix} \hat{\boldsymbol{x}}(t) \\ \hat{\boldsymbol{x}}_{\mathrm{d}}(t) \end{pmatrix} + \begin{pmatrix} \boldsymbol{b} \\ \mathbf{0} \end{pmatrix} u(t) + \boldsymbol{u}_{\mathrm{B}}(t) \\
\hat{y}(t) &= \begin{pmatrix} \boldsymbol{c}^{\mathrm{T}} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \hat{\boldsymbol{x}}(t) \\ \hat{\boldsymbol{x}}_{\mathrm{d}}(t) \end{pmatrix}
\end{aligned}
$$

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}u(t) + \boldsymbol{d}(t), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0$$
$$y(t) = \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}(t) + n(t)$$

$$\Downarrow$$

$$\begin{aligned}\Delta\dot{\boldsymbol{x}}(t) &= \boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t)\\ &= \dots\\ &= (\boldsymbol{A} - \boldsymbol{l}\boldsymbol{c}^{\mathrm{T}})\Delta\boldsymbol{x}(t) + \boldsymbol{d}(t) - \boldsymbol{l}n(t)\end{aligned}$$

Laplace-Bereich

$$\Delta\boldsymbol{X}(s) = \left(\mathbf{E}s - (\boldsymbol{A} - \boldsymbol{l}\boldsymbol{c}^{\mathrm{T}})\right)^{-1}\boldsymbol{D}(s) - \left(\mathbf{E}s - (\boldsymbol{A} - \boldsymbol{l}\boldsymbol{c}^{\mathrm{T}})\right)^{-1}\boldsymbol{l}N(s)$$
$$\Delta X(s) = \frac{1}{s - (a - lc)}D(s) - \frac{l}{s - (a - lc)}N(s)$$

$$\Downarrow$$

$$\boldsymbol{P}(t) = \mathrm{E}(\Delta\boldsymbol{x}(t)\Delta\boldsymbol{x}^{\mathrm{T}}(t)) = \mathrm{var}(\Delta\boldsymbol{x}(t))$$

$$\Uparrow$$

$$\dot{\hat{\boldsymbol{x}}}(t) = \boldsymbol{A}\hat{\boldsymbol{x}}(t) + \boldsymbol{b}u(t) + \boldsymbol{u}_{\mathrm{B}}(t), \quad \hat{\boldsymbol{x}}(0) = \hat{\boldsymbol{x}}_0$$
$$\hat{y}(t) = \boldsymbol{c}^{\mathrm{T}}\hat{\boldsymbol{x}}(t)$$

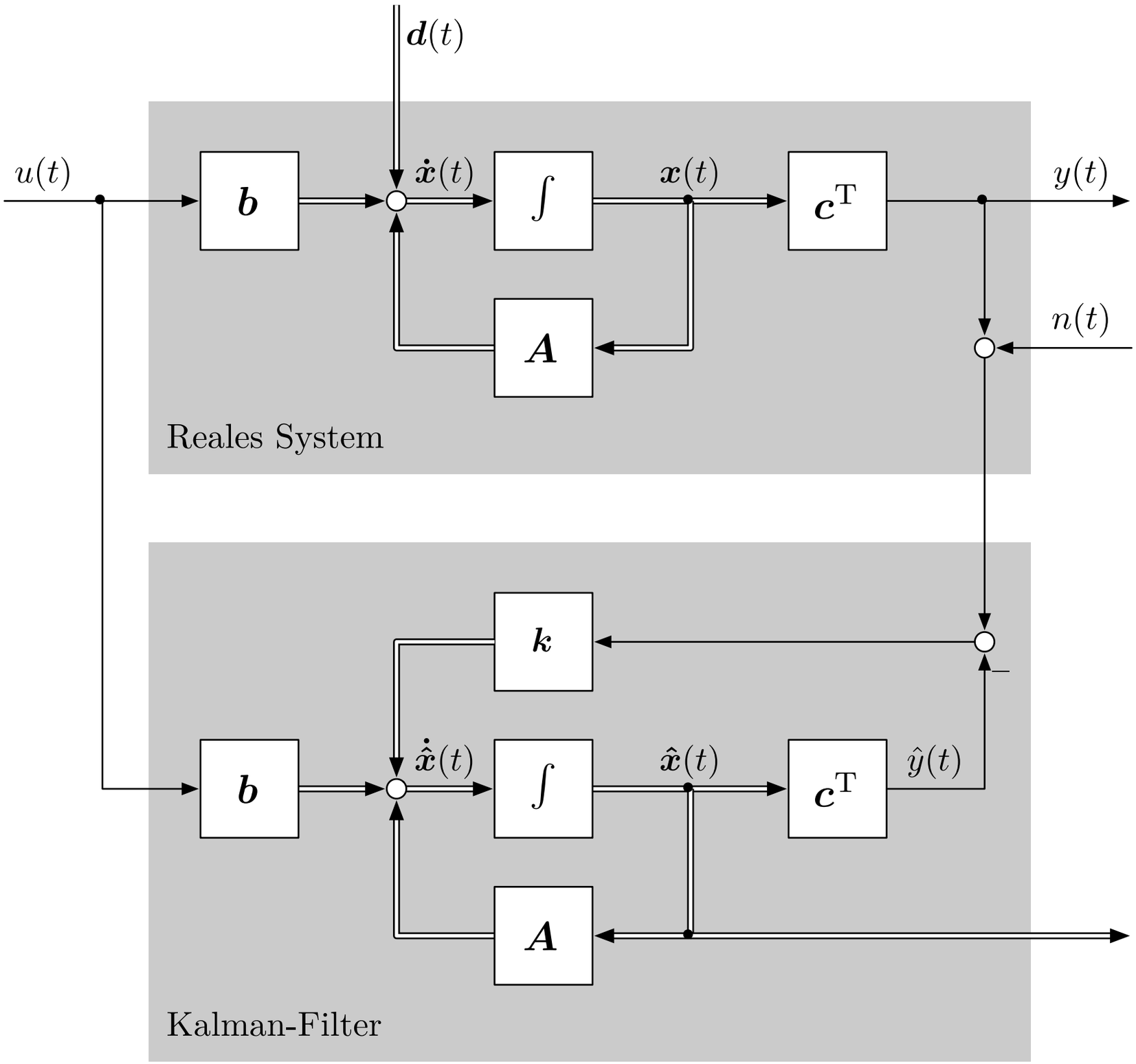
$$\boldsymbol{u}_{\mathrm{B}}(t) = \boldsymbol{l}(y(t) - \hat{y}(t)) = \boldsymbol{l}y(t) - \boldsymbol{l}\boldsymbol{c}^{\mathrm{T}}\hat{\boldsymbol{x}}(t)$$

$$\min_{\boldsymbol{K}} \boldsymbol{P}(t) = \min_{\boldsymbol{K}} \mathrm{E}(\Delta\boldsymbol{x}(t)\Delta\boldsymbol{x}^{\mathrm{T}}(t)) = \min_{\boldsymbol{K}} \mathrm{var}(\Delta\boldsymbol{x}(t))$$

$$\downarrow$$

$$\dot{\boldsymbol{P}}(t) = \boldsymbol{A}\boldsymbol{P}(t) + \boldsymbol{P}(t)\boldsymbol{A}^{\mathrm{T}} + \boldsymbol{Q} - \boldsymbol{P}(t)\boldsymbol{C}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{C}\boldsymbol{P}(t), \quad \boldsymbol{P}(0) = \mathrm{E}(\Delta\boldsymbol{x}(0)\Delta\boldsymbol{x}^{\mathrm{T}}(0))$$

$$\boldsymbol{K}(t) = \boldsymbol{P}(t)\boldsymbol{C}^{\mathrm{T}}\boldsymbol{R}^{-1}$$



$$\min_{\boldsymbol{K}} \boldsymbol{P}(t) = \min_{\boldsymbol{K}} \mathrm{E}(\Delta\boldsymbol{x}(t)\Delta\boldsymbol{x}^{\mathrm{T}}(t)) =$$

$$\dot{\boldsymbol{P}}(t) = \boldsymbol{A}\boldsymbol{P}(t) + \boldsymbol{P}(t)\boldsymbol{A}^{\mathrm{T}} + \boldsymbol{Q}$$

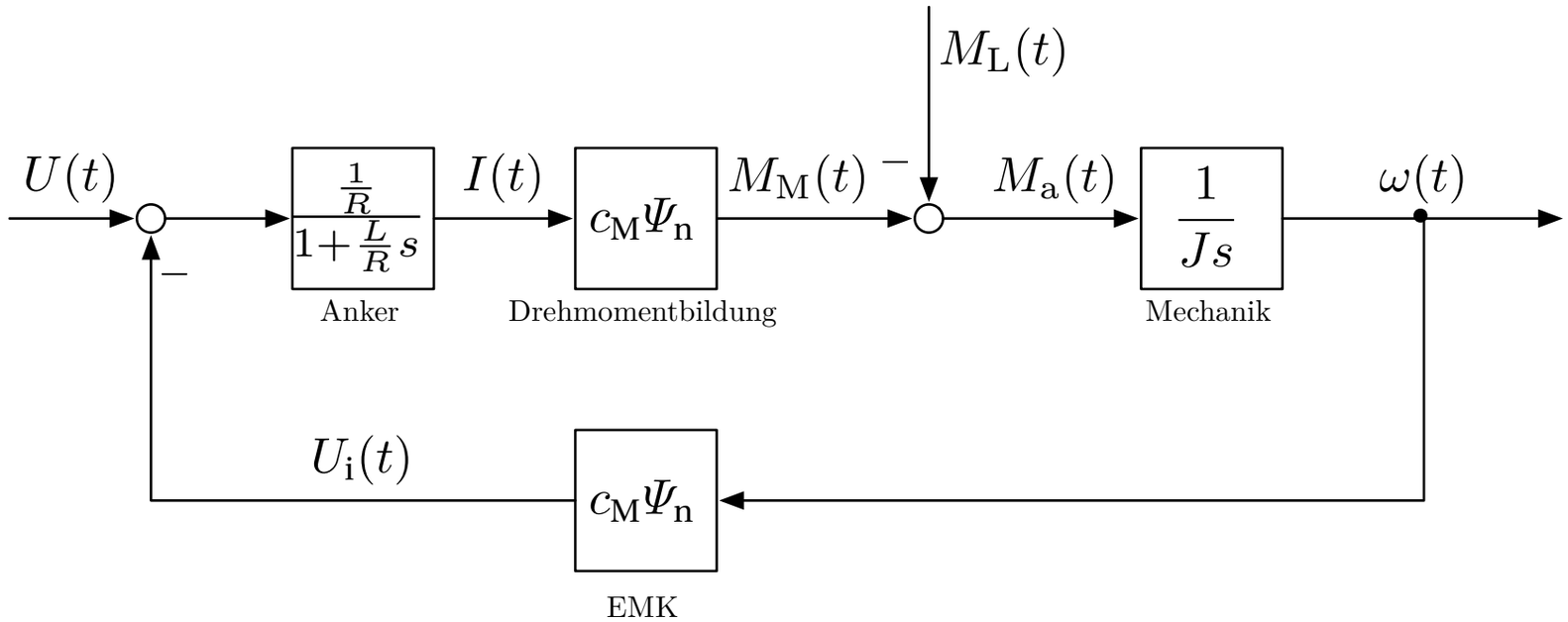
$$\boldsymbol{K}(t) = \boldsymbol{P}(t)\boldsymbol{C}^{\mathrm{T}}\boldsymbol{R}^{-1}$$

Nichtlineare Systeme:

Extended Kalman-Filter

$$\boldsymbol{A}(t) = \left.\frac{\mathrm{d}\boldsymbol{f}}{\mathrm{d}\boldsymbol{x}}\right|_{\hat{\boldsymbol{x}}(t),\boldsymbol{u}(t)}$$

$$\boldsymbol{C}(t) = \left.\frac{\mathrm{d}\boldsymbol{h}}{\mathrm{d}\boldsymbol{x}}\right|_{\hat{\boldsymbol{x}}(t)}$$

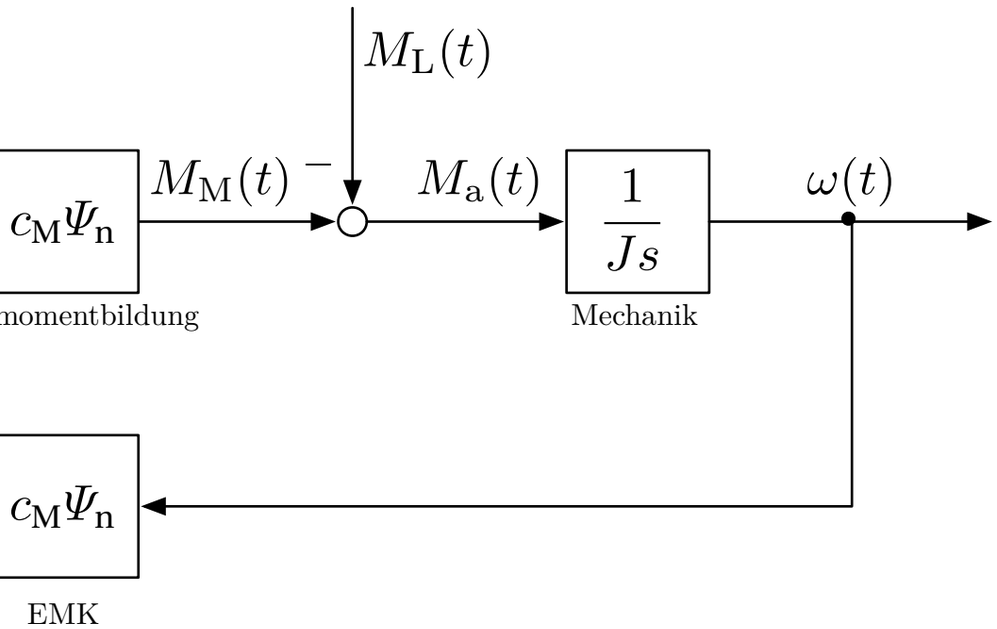


$$\Downarrow$$

$$\underbrace{\begin{pmatrix} \dot{I}(t) \\ \dot{\omega}(t) \\ \dot{M}_\mathrm{L}(t) \end{pmatrix}}_{\dot{\boldsymbol{x}}_\mathrm{S}(t)} = \underbrace{\begin{pmatrix} -\frac{R}{L} & -\frac{c_\mathrm{M}\Psi_\mathrm{n}}{L} & 0 \\ \frac{c_\mathrm{M}\Psi_\mathrm{n}}{J} & 0 & -\frac{1}{J} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} I(t) \\ \omega(t) \\ M_\mathrm{L}(t) \end{pmatrix}}_{\boldsymbol{A}_\mathrm{S}\boldsymbol{x}_\mathrm{S}(t)} + \underbrace{\begin{pmatrix} \frac{1}{L} \\ 0 \\ 0 \end{pmatrix} U(t)}_{\boldsymbol{b}_\mathrm{S}u(t)}$$

$$y(t) = \underbrace{\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} I(t) \\ \omega(t) \\ M_\mathrm{L}(t) \end{pmatrix}}_{\boldsymbol{c}_\mathrm{S}^\mathrm{T}\boldsymbol{x}_\mathrm{S}(t)} = I(t)$$

7 Testmodell Gleichstromnebenschlussmaschine



$$\underbrace{\begin{pmatrix} -\frac{c_\mathrm{M}\Psi_\mathrm{n}}{L} & 0 \\ 0 & -\frac{1}{J} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} I(t) \\ \omega(t) \\ M_\mathrm{L}(t) \end{pmatrix}}_{\boldsymbol{A}_\mathrm{S}\boldsymbol{x}_\mathrm{S}(t)} + \underbrace{\begin{pmatrix} \frac{1}{L} \\ 0 \\ 0 \end{pmatrix} U(t)}_{\boldsymbol{b}_\mathrm{S}u(t)}$$

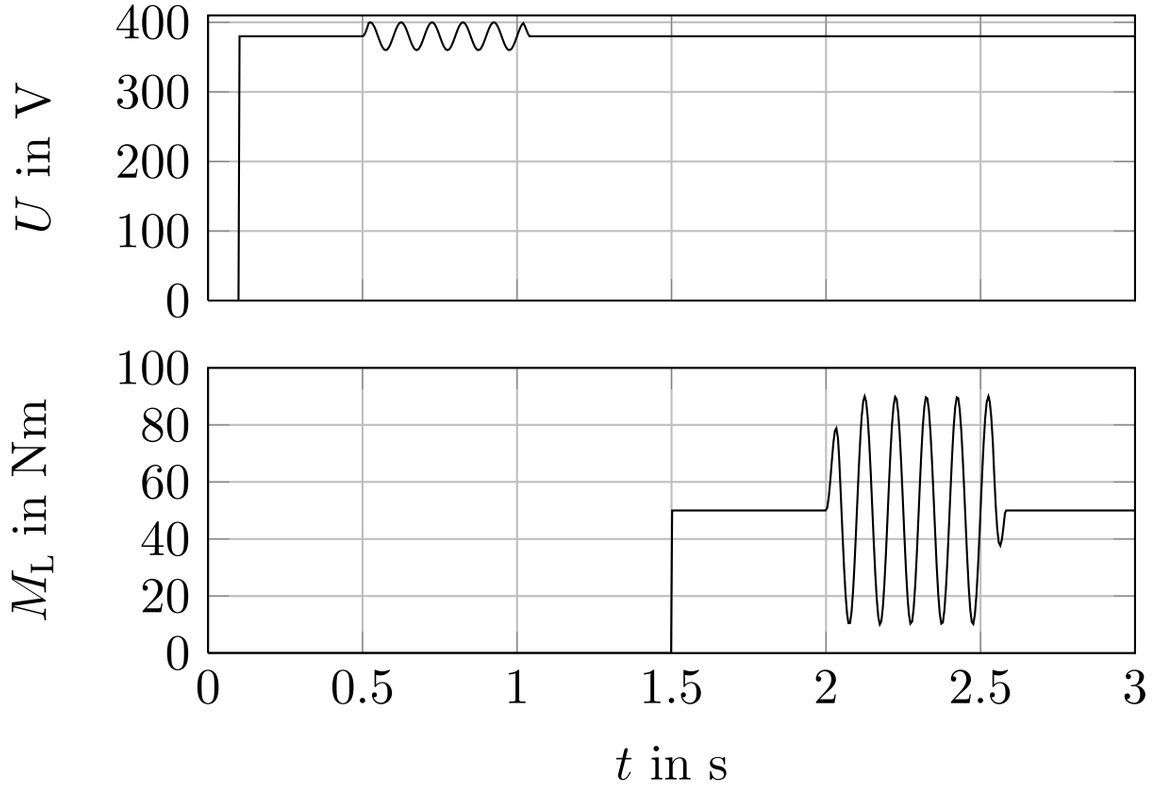
$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} I(t) \\ \omega(t) \\ M_\mathrm{L}(t) \end{pmatrix}}_{\boldsymbol{c}_\mathrm{S}^\mathrm{T}\boldsymbol{x}_\mathrm{S}(t)} = I(t)$$

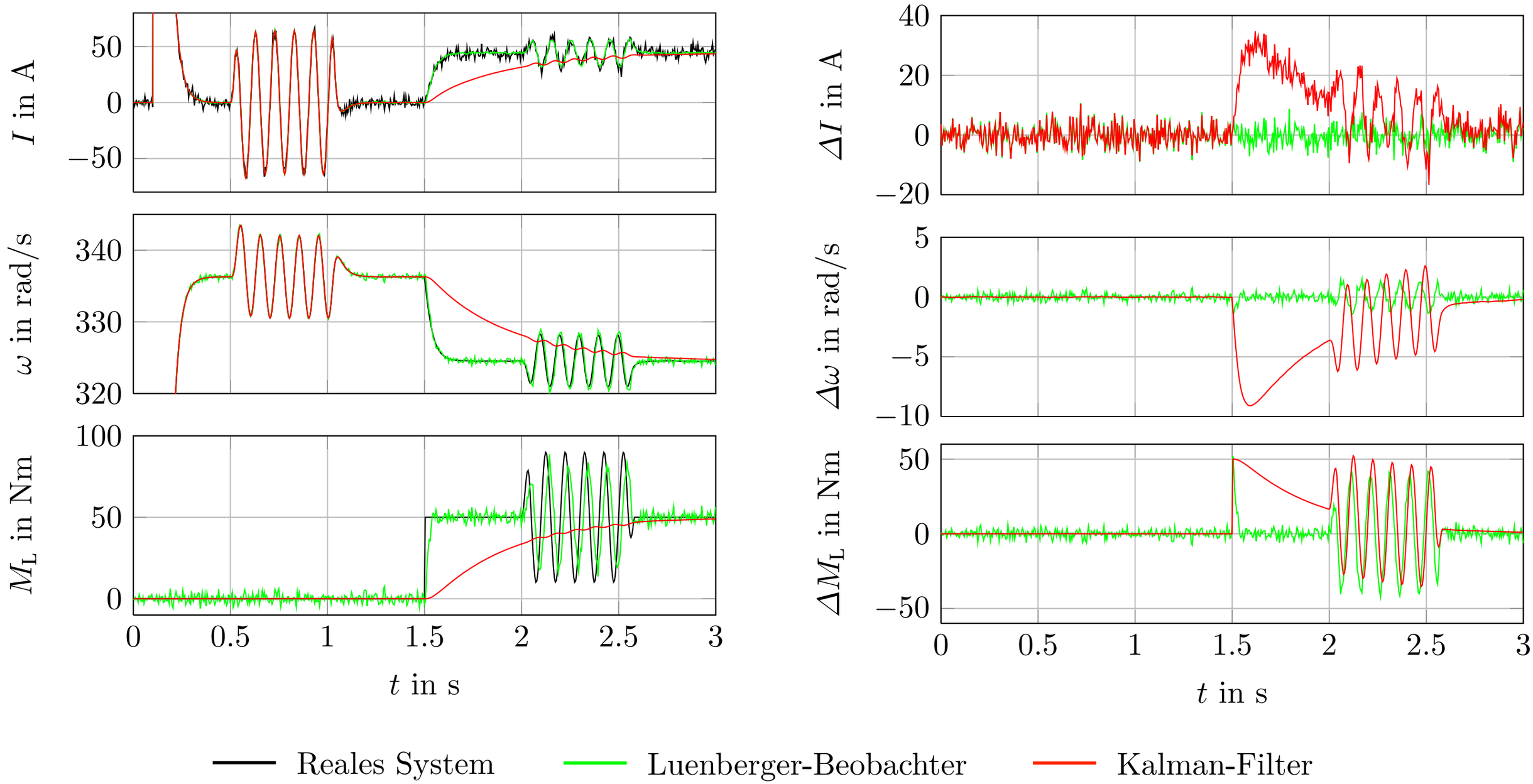
Luenberger-Beobachter:

$$\begin{aligned} \boldsymbol{\lambda}_\mathrm{S} &= 4\,\mathrm{eig}(\boldsymbol{A}_\mathrm{S}) \\ \boldsymbol{l}_\mathrm{S} &= \mathrm{place}\left(\boldsymbol{A}_\mathrm{S}^\mathrm{T}, \boldsymbol{c}_\mathrm{S}, \boldsymbol{\lambda}_\mathrm{S} + \begin{pmatrix} 0 \\ 0 \\ \lambda_{2,\mathrm{S}} \end{pmatrix}\right)^\mathrm{T} \\ &= \begin{pmatrix} 424,10 & -216,53 & 2243,67 \end{pmatrix}^\mathrm{T} \end{aligned}$$

Kalman-Filter:

$$\begin{aligned} \boldsymbol{k}_\mathrm{S} &= \mathrm{lqe}\left(\boldsymbol{A}_\mathrm{S}, \mathbf{E}, \boldsymbol{c}_\mathrm{S}^\mathrm{T}, \boldsymbol{Q}_\mathrm{S}, R\right)^\mathrm{T} \\ &= \begin{pmatrix} 10,15 & -1,50 & 10,15 \end{pmatrix}^\mathrm{T} \end{aligned}$$


400
300
200
100
0
U in V
100
80
60
40
20
0
M_L in Nm
0
0.5
1
1.5
2
2.5
3
t in s



9 Simulationsergebnisse, exaktes Modell

Luenberger-Beobachter	**Kalman-Filter**
Identische Struktur (stationär)	
Stationär genau mit Störgrößenmodell	
$\boldsymbol{l}$ aus Dynamikvorgabe	Dynamik nicht in Auslegung berücksichtigt
Rauschen nicht in Auslegung berücksichtigt	$\boldsymbol{k}$ aus System- und Messrauschen

➤ **Simulationen & Messungen zur Bestätigung benötigt!**

[1] Grewal, Mohinder S.; Andrews, Angus P.: *Kalman Filtering: Theory and Practice Using MATLAB*. Third Edition, Wiley-Verlag, 2008.

[2] Lunze, Jan: *Regelungstechnik 2: Mehrgrößensysteme, Digitale Regelung*. Springer-Verlag, 2008.

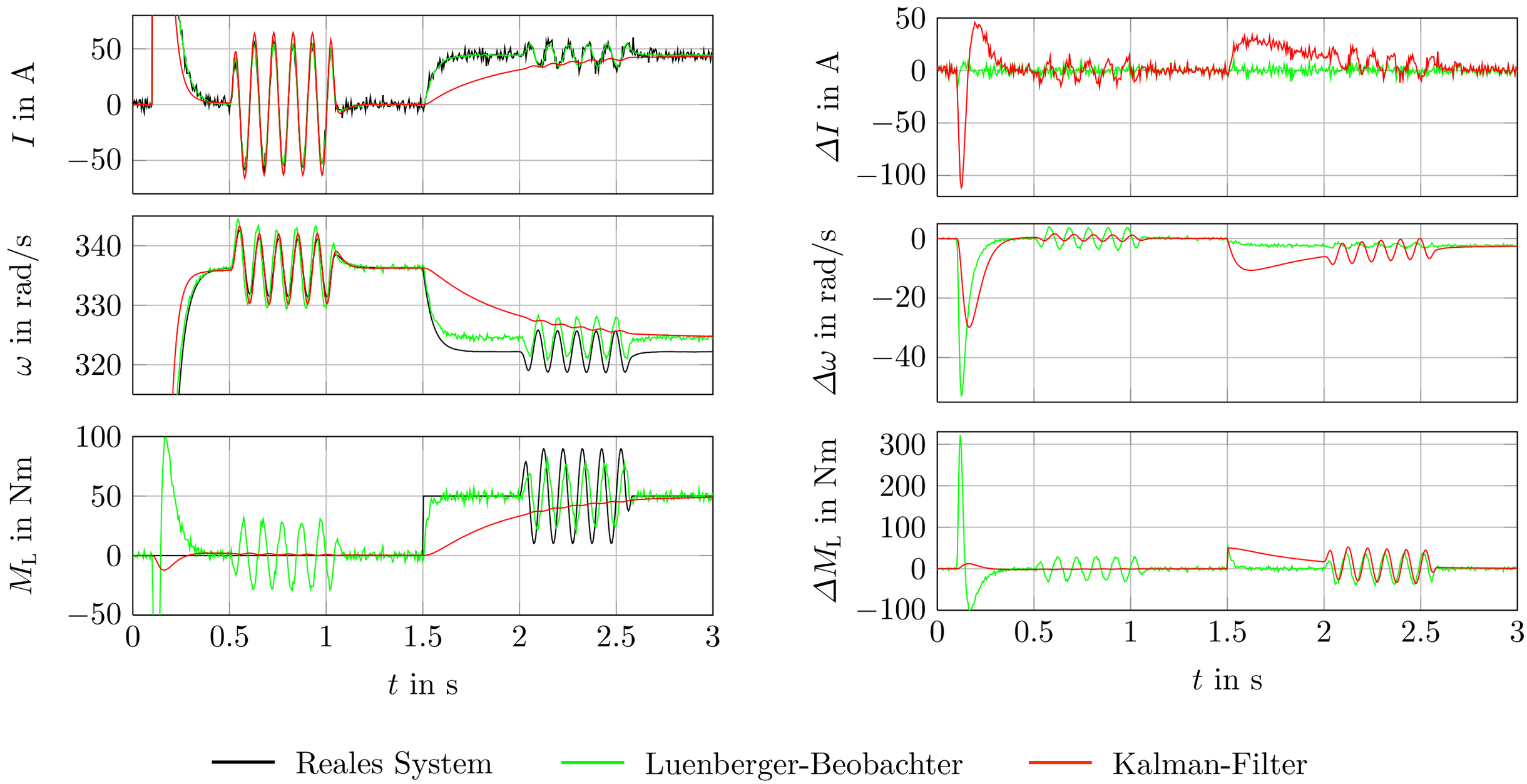
[3] Lunze, Jan: *Regelungstechnik 1: Systemtheoretische Grundlagen, Analyse und Entwurf einschleifiger Regelungen*. Springer-Verlag, 8. Auflage, 2010.

[4] Holzhüter, Prof. Dr. Thomas: *Zustandsregelung*. Skript, Fachhochschule Hamburg, Fachbereich Elektrotechnik und Informatik, 2009. `http://users.etech.haw-hamburg.de/users/holzhuet/zureg.pdf`, letzter Aufruf: 10.01.2010.

[5] Abbott, Dr. Jake: *Kalman Filter (Continous Time)*. Vorlesungsvideo zum Modul *State-Space Control Systems*, Department of Mechanical Engineering, The University of Utah. `https://www.youtube.com/watch?v=4Ef1RO06DnY`, letzter Aufruf: 10.01.2010.

[6] Föllinger, Otto: *Regelungstechnik – Einführung in die Methoden und ihre Anwendung*. 8. Auflage, Hüthig Buch Verlag Heidelberg, 1994.

[7] Ludyk, Günter: *Theoretische Regelungstechnik 2 – Zustandsrekonstruktion, optimale und nichtlineare Regelung*. Springer-Verlag, 1995.



Simulationsergebnisse, Parameteränderung $R' = 1{,}2\ R$ im realen Modell

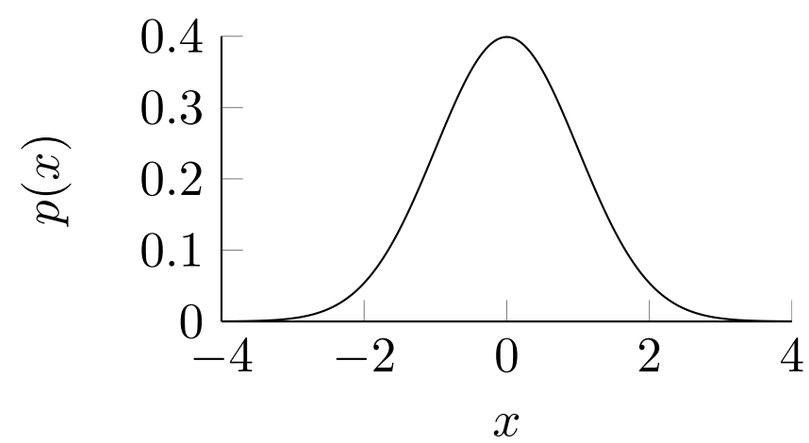
$$\mathrm{E}(\boldsymbol{d}) = \boldsymbol{0}$$
$$\mathrm{E}(\boldsymbol{n}) = \boldsymbol{0}$$

$$\mathrm{E}(\boldsymbol{d}(t)\boldsymbol{n}(\tau)^{\mathrm{T}}) = \mathrm{cov}(\boldsymbol{d}(t), \boldsymbol{n}(\tau)) = \boldsymbol{0}$$
$$\mathrm{E}(\boldsymbol{d}(t)\boldsymbol{x}_0^{\mathrm{T}}) = \mathrm{cov}(\boldsymbol{d}(t), \boldsymbol{x}_0) = \boldsymbol{0}$$
$$\mathrm{E}(\boldsymbol{n}(t)\boldsymbol{x}_0^{\mathrm{T}}) = \mathrm{cov}(\boldsymbol{n}(t), \boldsymbol{x}_0) = \boldsymbol{0}$$

stationäre, mittelwertfreie, weiße, gaußsche, unkorrelierte Rauschprozesse

$$\mathrm{E}(\boldsymbol{d}(t)\boldsymbol{d}(\tau)^{\mathrm{T}}) = \mathrm{cov}(\boldsymbol{d}(t), \boldsymbol{d}(\tau)) = \boldsymbol{Q}\delta(t-\tau) = \begin{cases} \boldsymbol{Q} & \text{für } t = \tau \\ \boldsymbol{0} & \text{sonst} \end{cases}$$

$$\mathrm{E}(\boldsymbol{n}(t)\boldsymbol{n}(\tau)^{\mathrm{T}}) = \mathrm{cov}(\boldsymbol{n}(t), \boldsymbol{n}(\tau)) = \boldsymbol{R}\delta(t-\tau) = \begin{cases} \boldsymbol{R} & \text{für } t = \tau \\ \boldsymbol{0} & \text{sonst,} \end{cases}$$



$$p(x) = \frac{1}{\sigma\sqrt{2\pi}}\mathrm{e}^{-\frac{x^2}{2\sigma^2}}$$

Geforderte Eigenschaften an das System- und Messrauschen